Constrained Linear Inversion of Satellite Ocean-Color Data

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ABSTRACT

An inversion methodology, based on least squares minimization, is proposed to retrieve spectral marine reflectance from top-of-atmosphere reflectance measurements in the visible and near infrared. The problem is first made linear by decomposing into principal components the additive contributions of the water body (the signal of interest) and of the atmosphere and surface (the perturbing signal), after subtraction of molecular effects and proper normalization. For realistic geometric and geophysical conditions, the two contributions can be described adequately by a few eigenvectors. This yields generally (i.e., for current satellite ocean-color sensors) an over-determined system of linear equations, in which the unknown parameters are the coefficients associated with the eigenvectors. The problem is ill conditioned, since the measurements are noisy, the signal of interest is small compared with the top-of-atmosphere reflectance, and some of the eigenvectors of the atmosphere/surface signal are correlated with those of the water-body signal. The system of linear equations is solved in the least squares sense using a regularization scheme, in which a regularization parameter is introduced to stabilize the solution. Once the coefficients are determined, they are used to reconstruct the water-body signal, basically the marine reflectance, and the perturbing signal. Performance is evaluated theoretically for the Sea-viewing Wide Field-of-view Sensor, using a comprehensive non-noisy simulated data set. The inversion scheme yields acceptable root-mean-squared errors of 0.0034, 0.00228, 0.00132, 0.00102, 0.00081, and 0.00021 on the retrieved water-body signal at 412, 443, 490, 510, 555, and 670 nm (Case 1 waters). Performance could be improved by using additional wavelengths in the near infrared, but more eigenvectors might be required to describe the atmospheric/surface signal.

Key words: Ocean color, remote sensing, aerosols, inversion, regularization, and atmospheric correction

1. INTRODUCTION

Algorithms developed to estimate the concentration of marine constituents (e.g., chlorophyll-a, dissolved organic matter, and sediments) from space\(^1\) try for correcting accurately effects of the atmosphere and surface on the measured top-of-atmosphere radiance. The procedure consists of estimating the aerosol radiance in the red and near infrared where the ocean can be considered black (i.e., totally absorbing), and extrapolating the estimated radiance to shorter wavelengths. The retrieved water-leaving radiance is then related to chlorophyll-a concentration or other biological variables using bio-optical models. This approach has been successful, and it is employed in the operational processing of data from major satellite ocean-color missions. In other algorithms\(^2\) aerosol properties and chlorophyll concentration are determined in a single step. Through systematic variation of candidate aerosol models, phytoplankton scattering, pigment concentration, and aerosol optical thickness, a best fit to the spectral top-of-atmosphere radiance (visible and near infrared) is obtained. The advantage of the single-step approach is its capability to handle both weakly and strongly absorbing aerosols\(^1\). The drawback is that convergence may not be achieved immediately in some cases, making it difficult to apply the algorithms to large satellite data sets.

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The two types of algorithms require, in particular, large look-up tables of aerosol optical properties, aerosol radiance, or top-of-atmosphere radiance. These tables are called internally as the atmospheric correction (standard procedure or non-linear optimization) is effected. The spectral-matching algorithms require a priori knowledge of the bio-optical model, even though this model can be general and encompass a large variety of water types. Changing the bio-optical model, which may be necessary depending on the region, requires regenerating the top-of-atmosphere radiance.

Other algorithms based on nonlinear regression methods have been developed, using perceptrons\textsuperscript{14} or ridge function fields\textsuperscript{15-16}. Depending on the selected class of regression models, but generally speaking, the advantage of regression approaches over the aforementioned algorithms is the rapidity of execution. Basically, the inverse model is first adjusted on a statistically significant synthetic data set, and then applied to actual data. However, because distributions of synthetic and actual data may differ from each other, due to radiometric calibration errors and radiation-transfer modeling uncertainties, the efficiency of regression techniques is strongly influenced by the overall noise distribution\textsuperscript{17}. Furthermore, like in the “spectral matching” algorithms, the adjustment is made using a specified bio-optical model, which may not represent the range of expected oceanic regimes.

In the method of Gross-Colzy et al.\textsuperscript{18, 19}, the satellite reflectance is first decomposed into principal components. The components sensitive to the ocean signal are then combined to retrieve the principal components of the marine reflectance. This allows a reconstruction of the marine reflectance and, therefore, an estimate of the marine reflectance. Neural network methodology is used to approximate the non-linear functions that relate the useful principal components of satellite reflectance to the principal components of marine reflectance. Keeping only the ocean-sensitive principal components of satellite reflectance reduces the influence of the atmosphere and surface, making the non-linear mapping accurate. By operating with principal components the algorithm is not be influenced by biases in radiometric calibration (only inter-band calibration needs to be accurate), making it well adapted to provide consistency across sensors and continuity in the quality of the marine reflectance\textsuperscript{20}.

The top-of-atmosphere reflectance in selected spectral bands may also be combined linearly, so that the atmosphere and surface effects are reduced substantially. This approach has been proposed to estimate chlorophyll-a concentration directly\textsuperscript{21}. The method requires appropriate modeling or decomposition of the perturbing effects and sufficient sensitivity of the linear combination to chlorophyll-a concentration. One advantage is that the bio-optical model can be changed easily, depending on the biological province considered. Another advantage is that the method is fast in application, and the resulting product is less noisy. Importantly, explicit knowledge of aerosol optical properties is avoided. The method has been extended to estimate the marine signal directly, without any assumption about the bio-optical properties of the water body\textsuperscript{22}, but remains to be tested on actual satellite imagery and evaluated against in-situ measurements.

Another scheme may be envisioned, in which the contributions of the atmosphere/surface (the perturbing signal) and water body (signal of interest) to the top-of-atmosphere radiance are decomposed into principal components. Only the important principal components are kept. This leads to a generally over-determined linear system of equations, with as unknowns the coefficients associated with the selected eigenvectors. The problem is solved by least squares minimization, but using the squared norm as regularization functional to stabilize the solution. First, we describe the approach and the procedure to determine the coefficients of the eigenvectors. Second, we evaluate theoretically, using simulated data, the errors on the retrieved marine signal. Finally we debate the advantages and drawbacks of the algorithm, and we conclude with a discussion of improvements and potential developments.

2. METHODOLOGY

2.1 Problem

Instead of using radiance $L$, we use reflectance $R$ defined as $R = \frac{\pi L}{F_0 \cos \theta_0}$, where $F_0$ is the extraterrestrial solar irradiance and $\theta_0$ is the solar zenith angle. After correction for gaseous absorption and effects of the molecular atmosphere, the top-of atmosphere reflectance becomes:
\[
R_p(\lambda) = R(\lambda) - R_0(\lambda) + Rw(\lambda)T_m(\lambda)Ta(\lambda)
\]  
(1)

where \( \lambda \) is the measurement wavelength, \( R(\lambda) \) is the top-of-atmosphere reflectance when the water body is black and the atmosphere contains aerosols and molecules, and \( R_0(\lambda) \) is the top-of-atmosphere reflectance when the water body is black and the atmosphere contains only molecules, \( Rw(\lambda) \) is the water-body reflectance, and \( T_m(\lambda) \) and \( Ta(\lambda) \) are diffuse transmittances due to molecules and aerosols, respectively. Equation (1) can be re-written conveniently in vector form as

\[
z = x + y
\]  
(2)

where \( x, y, \) and \( z \) are \( \mathbb{R}^k \) vectors representing the variables \( R - R_0, RwTmTa, \) and \( Rp \) at \( k \) measurement wavelengths.

Let \( (x_1,y_1,z_1),\ldots,(x_n,y_n,z_n) \) be an ensemble of size \( n \), obtained by simulation, with \( z_i = x_i + y_i \). The principal component analysis of \( \{x_i\} \) and \( \{y_i\} \) projects \( x \) and \( y \) in \( p \)- and \( q \)-dimensional spaces:

\[
x = \sum a_ie_i, \quad i = 1,2,\ldots,p
\]  
(3a)

\[
y = \sum b_if_j, \quad j = 1,2,\ldots,q
\]  
(3b)

where \( e_i \) and \( f_j \) are the principal components (eigenvectors of the covariance matrices) and \( a_i \) and \( b_j \) the associated coefficients.

Let \( P \) denotes the matrix of size \( pxq \) defined by \( [e_1,e_2,\ldots,e_p,f_1,f_2,\ldots,f_q] \) and let \( u \) denote the vector of coefficients \( u := (a_1,a_2,\ldots,a_p,b_1,b_2,\ldots,b_q) \). The following model is obtained, in matrix form:

\[
z = Pu
\]  
(4)

The problem, therefore, is to retrieve \( u \) from \( z \), knowing \( P \).

2.2 Solution

Given \( z \), the best solution in the least-square sense is

\[
\hat{u} = (P^TP)^{-1}P^Tz
\]  
(5)

where \( P^T \) is the transpose of \( P \).

The solution, however, may be unstable (e.g., \( z \) is noisy, the model in not perfect, some \( e_i \) and \( f_j \) may be correlated), but stability may be achieved by regularization. In the Tikhonov regularization scheme, the solution is obtained, not by minimizing the norm of the residual, but by minimizing \( \|z - Pu\| + \alpha \|u\|^2 \) where \( \alpha \) is a strictly positive scalar called the regularization parameter. The solution is then given by

\[
\hat{u} = (P^TP + \alpha I)^{-1}P^Tz
\]  
(6)

where \( I \) is the identity matrix.

Once \( \hat{u} := (\hat{u}_1,\hat{u}_2,\ldots,\hat{u}_{p+q}) \) is obtained, the contributions \( x \) and \( y \) can be reconstructed:

\[
x = \sum \hat{u}_ie_i, \quad i = 1,2,\ldots,p
\]  
(7a)

\[
y = \sum \hat{u}_{p+i}f_i, \quad i = 1,2,\ldots,q
\]  
(7b)
3. RESULTS

The Top-of-atmosphere reflectance was simulated at the SeaWiFS wavelengths, i.e., 412, 443, 490, 510, 555, 670, 765, and 865 nm, for a wide range of geometric and geophysical conditions, i.e., Sun zenith angle between 0 and 60 degrees, view zenith angle between 0 and 60 degrees, relative azimuth angle between 0 and 180 degrees, wind speed between 0.1 and 15 m s\(^{-1}\), aerosol optical thickness at 550 nm between 0.01 and 0.8, mixtures of continental/urban, maritime/urban, and maritime/urban aerosols, biomass-burning aerosols, and dust-like aerosols, and chlorophyll-a concentration from 0.03 to 30 mg m\(^{-3}\) (Case 1 waters). The model of Vermote et al.\(^{23}\) was used to simulate the radiative transfer in the ocean-surface-atmosphere system, and the water-body reflectance was simulated according to Morel and Maritorena\(^{24}\).

Only cases for which sun glint reflectance was less than 0.04 were selected, resulting in 15,436x3 cases of continental/urban, maritime/urban, and maritime/continental mixtures and 5,000x2 cases of biomass-burning and dust-like aerosols, i.e., a total of 56,308 cases. In practice, the RT code was run twice for each case, once with aerosols and a non-black water body, and once without aerosols and a black water body. The difference between the top-of-atmosphere reflectance from the two runs yielded the corrected top-of-atmosphere reflectance to invert. Examples of \(x\), \(x\), and \(y\) spectra are displayed in Figure 1, showing the relative importance of \(x\) and \(y\) in \(z\).

A principal component analysis of the \(x\) and \(y\) vector ensembles revealed that each \(x\) and \(y\) could be accurately represented by the first 4 principal components (Table 1), which restituted 99.998 and 99.992% of the variance, respectively. Using these principal components, \(z\) is well reconstructed, as shown in Figures 2 and 3. For \(x\), the root-mean-squared difference between reconstructed and actual values is 0.00006, 0.00012, 0.00007, 0.00008, 0.00018, 0.00011, 0.00008, and 0.00007 at 412, 443, 490, 510, 555, 670, 765, and 865 nm (Table 2). For \(y\), the values are 0.00003, 0.00005, 0.00001, 0.00003, 0.00003, and 0.00005 at 412, 443, 490, 510, 555, and 670 nm (Table 3). Some eigenvectors of \(x\), however, are correlated with those of \(y\) (Table 4). Most notably, \(e_2\) is strongly correlated with \(f_1\) and \(f_2\), \(e_3\) to \(f_3\), \(e_4\) to \(f_4\), and \(e_5\) to \(f_5\). One expects, therefore, the solution to be sensitive to the \(x\) and \(y\) models and to noise in \(z\), pointing to the necessity of applying some constraint to the solution and make it stable.

In view of the results of the principal component analysis, the Tikhonov regularization scheme was applied to solving the problem \(z = Pu\) with \(P := [e_1,e_2,e_3,e_4,f_1,f_2,f_3,f_4]\), \(u := (a_1,a_2,a_3,a_4, b_1,b_2,b_3,b_4),\) i.e., \(x\) and \(y\) each described by 4 eigenvectors, and \(z\) in \(R^8\) (measurements at the 8 SeaWiFS wavelengths). The regularization parameter \(\alpha\) was varied from 0.0001 to 0.01, and the best results were obtained for \(\alpha = 0.005\). Figure 4 displays the retrieved \(x\) versus the actual \(x\). All the cases are plotted in the left panels, and the average error and standard deviation in bins (10 bins in the range of variability of \(x\)) are plotted in the right panels. The atmosphere/surface signal is well retrieved at the longer wavelengths, i.e., 555, 670, 765, and 865 nm, but performance is degraded as wavelength decreases. The error also increases when \(x\) is large (high aerosol loading). The retrieval of \(y\) is not as good as for \(x\), as evidenced in Figure 5, left, which shows a large scatter in retrieved \(y\) versus actual \(y\). The average error and standard deviation in bins, however, are acceptable (Figure 5, right). The statistical performance for \(y\) is consistent with the errors in the atmosphere/surface signal, since \(x\) is generally much larger than \(y\).

Overall (i.e., all cases considered), the bias is negligible at all wavelengths, and the root-mean-squared error is 0.0034, 0.0023, 0.0013, 0.0010, 0.0008, and 0.0002 at 412, 443, 490, 510, 555, and 670 nm (Table 5). In the presence of biomass burning or dust-like aerosols, however, the root-mean-squared error is higher, especially in the blue (e.g., 0.0046 and 0.0057 at 412 nm, respectively). One should note that the simulations included cases of very high aerosol loading (i.e., aerosol optical thickness of 0.8 at 550 nm). Such situations are usually not encountered in the open ocean\(^{25}\), where most values remain below 0.3. The algorithm performance is better at lower optical thickness, as indicated in Figure 6, which shows the root-mean squared error as a function of aerosol optical thickness. At 412 nm, for instance, the root-mean-squared error is decreased from 0.0052 to 0.0027 (i.e., by a factor of about 2) when the optical thickness is decreased from 0.8 to 0.3.
4. CONCLUSIONS

Decomposing the atmosphere/surface signal and the water-body signal in principal components yields a system of linear equations in which the unknowns are the coefficients associated with the eigenvectors describing the individual signals. The inverse problem is ill posed, however, in the sense that the linear models of the contributing signals are not exact, the measurements are noisy, and some of the eigenvectors of the atmosphere/surface signal are correlated with those of the water-body signal. To solve the problem, a regularization scheme is employed, which uses the squared norm as regularization functional (Tikhonov regularization). The constrained linear inversion scheme, when applied to SeaWiFS simulated data, yields acceptable root-mean-squared errors of 0.0034, 0.00228, 0.00132, 0.00102, 0.00081, and 0.00021 on the retrieved water-body signal (i.e., reflectance multiplied by atmospheric transmittance) at 412, 443, 490, 510, 555, and 670 nm. The root-mean squared errors decrease with aerosol optical thickness, for example from 0.0052 to 0.0027 at 412 nm, when aerosol optical thickness is decreased from 0.8 to 0.3, and errors are the largest when aerosols are absorbing (biomass-burning and dust-like).

Performance could be improved by using additional wavelengths in the near infrared, but more eigenvectors (i.e., more coefficients to determine) might be required to describe accurately the atmospheric/surface signal. Additional constraints may be introduced, which take into account the domain of the solution. The simulated marine reflectance did not include biological noise (due to phytoplankton type). Adding this noise would make the simulated data more realistic, and the representation of the marine signal with only four eigenvectors would remain adequate. Importantly, stability of the solution needs to be tested with noise in the measurements, and performance should be evaluated using actual satellite imagery and match-up data sets.

One advantage of the method, compared with standard and spectral-matching techniques, is that no look-up tables of geometry-dependent aerosol optical properties or top-of-atmosphere reflectance need to be called and, therefore, created. Another advantage is the rapid execution of the algorithm. However, like spectral matching or nonlinear regression techniques, the method does require a priori knowledge of the bio-optical model or some assumption about the water body. The method was developed for Case I waters, for which the reflectance in the near infrared is practically negligible. It could be extended to Case II waters, but this would require selecting longer wavelengths (above 1000 nm), for which Case II waters are black. In addition, the linear model of marine reflectance would likely require additional eigenvectors, therefore more equations and measurements. This possibility and the potential improvements mentioned above, including evaluation activities, will be the subject of future work.

ACKNOWLEDGMENTS

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REFERENCES

Figure 1. Examples of $z$, $x$ and $y$ spectra: (Top) TOA signal, $z$; (Middle) Atmospheric/surface signal, $x$; (Bottom) Water-body signal, $y$.

Table 1. Principal component analysis of the atmospheric/surface signal, $x$, and of the water-body signal, $y$. The first 4 principal components are selected to represent $x$ and $y$.

<table>
<thead>
<tr>
<th>Atmospheric/Surface contribution, $x$</th>
<th></th>
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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Principal component:</td>
<td>PC1</td>
<td>PC2</td>
<td>PC3</td>
<td>PC4</td>
<td>PC5</td>
<td>PC6</td>
<td>PC7</td>
<td>PC8</td>
</tr>
<tr>
<td>Standard deviation:</td>
<td>0.0708</td>
<td>0.0131</td>
<td>0.00438</td>
<td>0.000389</td>
<td>0.000245</td>
<td>0.000142</td>
<td>7.66e-05</td>
<td>5.23e-05</td>
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<tr>
<td>Proportion of Variance:</td>
<td>0.9630</td>
<td>0.0332</td>
<td>0.00369</td>
<td>0.000030</td>
<td>0.000010</td>
<td>0.000000</td>
<td>0.00e+00</td>
<td>0.00e+00</td>
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<td>Cumulative Proportion:</td>
<td>0.9630</td>
<td>0.9963</td>
<td>0.99995</td>
<td>0.999980</td>
<td>0.999990</td>
<td>1.000000</td>
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<table>
<thead>
<tr>
<th>Water-body contribution, $y$</th>
<th></th>
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<th></th>
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</tr>
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<tbody>
<tr>
<td>Principal component:</td>
<td>PC1</td>
<td>PC2</td>
<td>PC3</td>
<td>PC4</td>
<td>PC5</td>
<td>PC6</td>
<td></td>
<td></td>
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<tr>
<td>Standard deviation:</td>
<td>0.00944</td>
<td>0.00249</td>
<td>0.00131</td>
<td>0.000253</td>
<td>8.36e-05</td>
<td>2.16e-05</td>
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<tr>
<td>Proportion of Variance:</td>
<td>0.91749</td>
<td>0.06402</td>
<td>0.01776</td>
<td>0.000660</td>
<td>7.00e-05</td>
<td>0.00e+00</td>
<td></td>
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<tr>
<td>Cumulative Proportion:</td>
<td>0.91749</td>
<td>0.98150</td>
<td>0.99926</td>
<td>0.999920</td>
<td>1.00e+00</td>
<td>1.00e+00</td>
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<td></td>
</tr>
</tbody>
</table>
Figure 2. Reconstructed versus actual $x$ for the 56,308 cases of the simulated ensemble; only the 4 main principal components are used in the reconstruction of $x$.

Table 2. Comparison statistics for reconstructed versus actual $x$. For each wavelength, correlation coefficient squared ($r^2$), mean difference (bias), and root-mean-squared difference (rms) are given.

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>$r^2$</th>
<th>Bias</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>412</td>
<td>0.999992</td>
<td>-5.80633e-09</td>
<td>6.01015e-05</td>
</tr>
<tr>
<td>443</td>
<td>0.999974</td>
<td>3.06661e-08</td>
<td>0.000128342</td>
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<tr>
<td>490</td>
<td>0.999987</td>
<td>-7.56543e-08</td>
<td>6.70051e-05</td>
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<td>510</td>
<td>0.999996</td>
<td>-1.80984e-07</td>
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<td>555</td>
<td>0.999954</td>
<td>-5.46175e-08</td>
<td>0.000183667</td>
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<td>670</td>
<td>0.999977</td>
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<td>0.000110643</td>
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<tr>
<td>765</td>
<td>0.999980</td>
<td>1.28159e-07</td>
<td>8.14040e-05</td>
</tr>
<tr>
<td>865</td>
<td>0.999988</td>
<td>-1.57875e-07</td>
<td>7.18548e-05</td>
</tr>
</tbody>
</table>
**Figure 3.** Reconstructed versus actual $y$ for the 56,308 cases of the simulated ensemble; only the 4 main principal components are used in the reconstruction of $y$.

**Table 3.** Comparison statistics for reconstructed versus actual $x$. For each wavelength, correlation coefficient squared ($r^2$), mean difference (bias), and root-mean-squared difference (rms) are given.

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>$r^2$</th>
<th>Bias</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>412</td>
<td>0.999955</td>
<td>0.999989</td>
<td>1.22384e-09</td>
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<td>443</td>
<td>0.999967</td>
<td>0.999934</td>
<td>1.29119e-08</td>
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<tr>
<td>490</td>
<td>0.999983</td>
<td>0.999967</td>
<td>2.53352e-08</td>
</tr>
<tr>
<td>510</td>
<td>0.999824</td>
<td>0.999649</td>
<td>2.58984e-10</td>
</tr>
<tr>
<td>555</td>
<td>0.999899</td>
<td>0.999797</td>
<td>4.61148e-09</td>
</tr>
<tr>
<td>670</td>
<td>0.998449</td>
<td>0.996900</td>
<td>-8.01196e-09</td>
</tr>
</tbody>
</table>
Table 4. Correlation (i.e., cosine of angle) between eigenvectors $e_i$ and $f_j$.

<table>
<thead>
<tr>
<th>$e_i$, $f_j$:</th>
<th>-0.603212</th>
<th>$e_2$, $f_1$:</th>
<th>0.923051</th>
<th>$e_3$, $f_1$:</th>
<th>-0.471825</th>
<th>$e_4$, $f_1$:</th>
<th>0.0474317</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_1$, $f_2$:</td>
<td>-0.743702</td>
<td>$e_2$, $f_2$:</td>
<td>-0.111731</td>
<td>$e_3$, $f_2$:</td>
<td>0.785511</td>
<td>$e_4$, $f_2$:</td>
<td>0.0212801</td>
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<tr>
<td>$e_1$, $f_3$:</td>
<td>-0.0733225</td>
<td>$e_2$, $f_3$:</td>
<td>-0.0722633</td>
<td>$e_3$, $f_3$:</td>
<td>-0.338039</td>
<td>$e_4$, $f_3$:</td>
<td>-0.758633</td>
</tr>
<tr>
<td>$e_1$, $f_4$:</td>
<td>-0.200459</td>
<td>$e_2$, $f_4$:</td>
<td>-0.302401</td>
<td>$e_3$, $f_4$:</td>
<td>0.195714</td>
<td>$e_4$, $f_4$:</td>
<td>-0.00377064</td>
</tr>
</tbody>
</table>

Figure 4. Retrieved $x$ versus actual $x$. (Left: all points; Right: by bins.)
Figure 5. Retrieved y versus actual y. (Left: all points; Right: by bins.)

Table 5. Overall performance statistics for y. For each wavelength, correlation coefficient squared ($r^2$), mean difference (bias), and root-mean-squared difference (rms) are given.

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>$r^2$</th>
<th>Bias</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>412</td>
<td>0.818552</td>
<td>-5.09920e-06</td>
<td>0.00343865</td>
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<tr>
<td>443</td>
<td>0.852188</td>
<td>-3.45526e-06</td>
<td>0.00228534</td>
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<td>490</td>
<td>0.852291</td>
<td>-1.95842e-06</td>
<td>0.00131932</td>
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<td>510</td>
<td>0.744451</td>
<td>-1.40479e-06</td>
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<td>670</td>
<td>0.951425</td>
<td>3.13514e-07</td>
<td>0.000208212</td>
</tr>
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Figure 6. Root-mean-squared (rms) error on retrieved $y$ as a function of aerosol optical thickness at 550 nm.